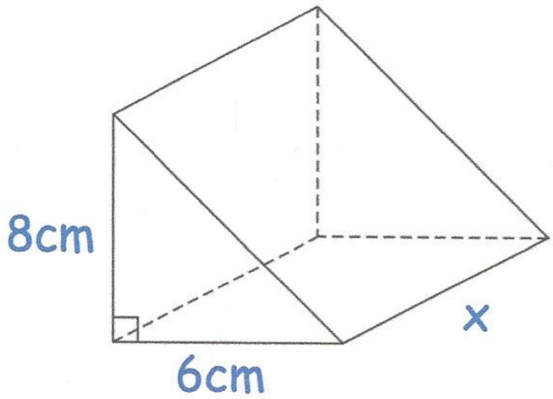
26

The diagram shows a solid triangular prism.



The prism is made from wood and has a mass of 643.8g
The density of wood is 1.85g/cm³

Calculate the length of the prism.

$v = \frac{m}{d}$ $\frac{643.8}{1.85} = 348 \text{ cm}^3$

$24 \times x = 348$

14.5 ....cm

(4)

27

Timothy weighs the mass of some oranges, in grams.

The table shows some information about his results.

Mass	Frequency	midpoint	fx
$20 < m \leq 25$	12	22.5	270
$25 < m \leq 30$	24	27.5	660
$30 < m \leq 35$	17	32.5	552.5
$35 < m \leq 40$	15	37.5	562.5
$40 < m \leq 45$	4	42.5	170
	+ 72		+ 2215

Work out an estimate for the mean mass of an orange.

$2215 \div 72$

$30.763\dot{8}$ ....grams

(4)

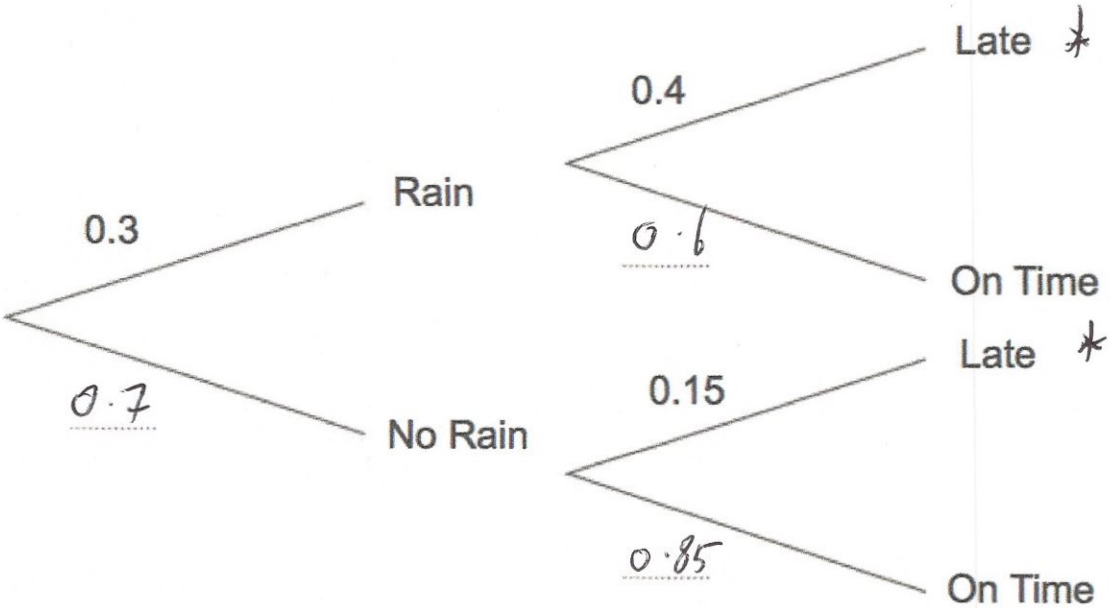
28

In a small village, one bus arrives a day.

The probability of rain in the village is 0.3.

If it rains, the probability of a bus being late is 0.4.
If it does not rain, the probability of a bus being late is 0.15.

(a) Complete the tree diagram



(2)

(b) Work out the number of days the bus should be late over a period of 80 days.

$P(RL) = 0.3 \times 0.4 = 0.12$

$P(NR\ L) = 0.7 \times 0.15 = 0.105$

$P(\text{Late}) = 0.225$

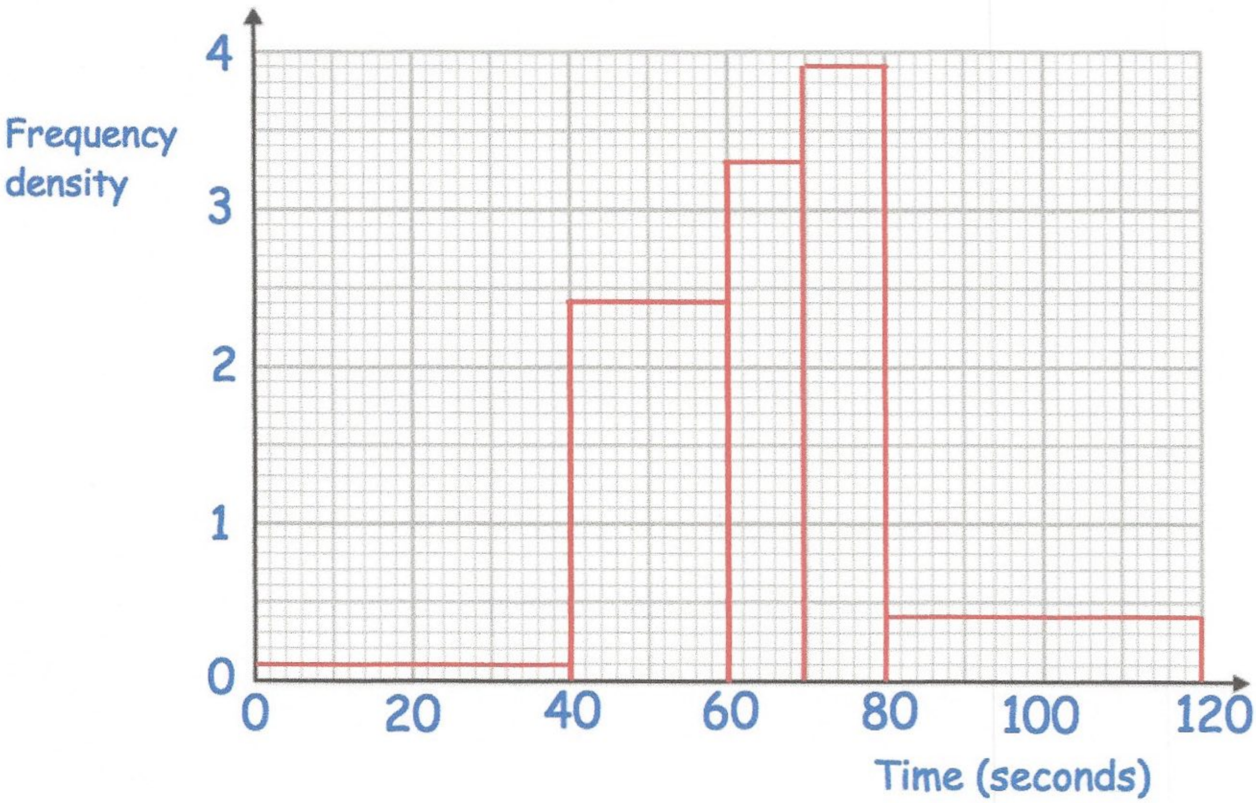
80×0.225

18 days

(3)

数 29

The histograms shows information about the time taken by 140 students to complete a puzzle.



(a) Complete this frequency table.

Time, t seconds	Frequency
$0 < t \leq 40$	4
$40 < t \leq 60$	48
$60 < t \leq 70$	33
$70 < t \leq 80$	39
$80 < t \leq 120$	16

20×2.4

10×3.9

(2)

(b) Calculate an estimate of the median.

70^{th} Value

$60 + \frac{18}{33} \times 10$

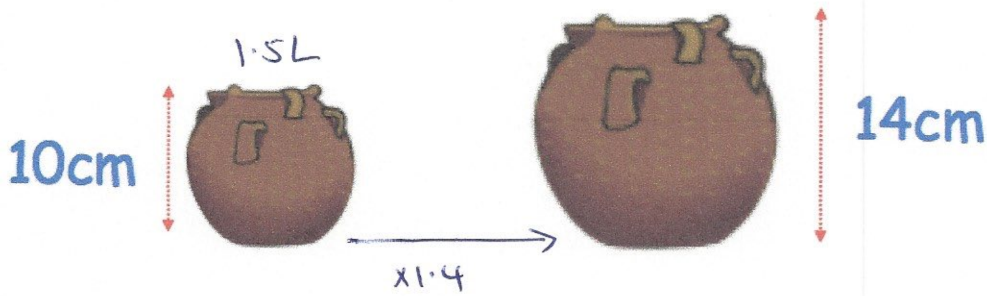
65.455 seconds

(3)

32. Mrs Hampton is potting plants.
She is using two mathematically similar pots, the smaller is 10cm tall and the larger 14cm tall.

She has two bags of soil, each containing 30 litres of soil.

With the first bag, Mrs Hampton fills 20 small pots using all of the soil in the bag.



How many large pots can be filled completely using the second bag of soil?

$30 \div 20 = 1\cdot5$ litres

$1\cdot5 \times 1\cdot4^3 = 4\cdot116$ litres

$30 \div 4\cdot116 = 7\cdot28\ldots$

..........7..........
(5)

---

33. Declan ran a distance of 200m in a time of 26.2 seconds.

The distance of 200m was measured to the nearest 10 metres.
The time of 26.2 was measured to the nearest tenth of a second.

Work out the upper bound for Declan's average speed.

$S = \frac{d}{t}$

$\text{Max } S = \frac{\text{Max } d}{\text{Min } t} = \frac{205}{26\cdot15}$

..........7·839..........m/s
(2)

32

Factorise fully

$$w^2y + wy^2$$

$wy(w+y)$

$wy(w+y)$ **(2)**

33

~~60~~. (a) Factorise $x^2 + 14x - 51$

$(x + 17)(x - 3)$

(2)

(b) Factorise $2w^2 - 9w + 4$

$(w - 4)(2w - 1)$

(2)

(c) Factorise $x^2 - 121$

$(x + 11)(x - 11)$

(2)

34

(a) Solve $y^2 + 9y + 2 = 8y + 58$

$y^2 + y - 56 = 0$

$(y + 8)(y - 7) = 0$

$y = -8$ or $y = 7$

$y = -8$ or $y = 7$

(2)

~~(2)~~

(b) Solve $5x^2 + 19x - 4 = 0$

$(5x - 1)(x + 4) = 0$

$5x = 1$ or $x = -4$

$x = 1/5$

....

(2)

35

Solve the equation $x^2 - 2x - 9 = 0$

Give your answers to two decimal places.

$a = 1$

$b = -2$

$c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = $ 4.16 or $x = $ -2.16

(3)

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-9)}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{4 - (-36)}}{2}$$

$x = \frac{2 + \sqrt{40}}{2}$ or $x = \frac{2 - \sqrt{40}}{2}$

36

The nth term of a sequence is $4n - 7$

(a) Write down the first three terms of the sequence.

1st term ....-3...., 2nd term ....1...., 3rd term ....5....

(2)

(b) What is the difference between the 150th and 151st terms?

4

(1)

The last term of this sequence is 393.

(c) How many terms are there in this sequence?

$4n - 7 = 393$

$4n = 400$

$n = 100$

100

(2)

37

Here are the first 5 terms of a quadratic sequence

9 17 29 45 65

Find an expression, in terms of n, for the nth term of this quadratic sequence.

$a+b+c$ 9 17 29 45 65

$3a+b$ 8 12 16 20

$2a$ 4 4 4

$2a = 4$

$a = 2$

$3a+b = 8$

$6+b = 8$

$b = 2$

$a+b+c = 9$

$2+2+c = 9$

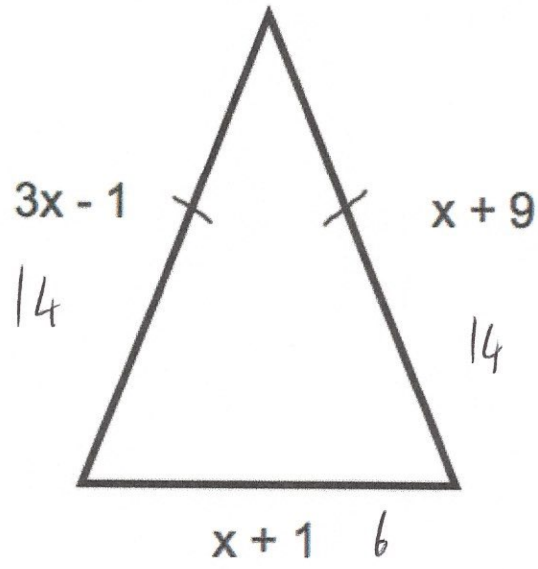
$c = 5$

$2n^2 + 2n + 5$

(3)

38

Shown below is an isosceles triangle. Each side is measured in centimetres.



Find the perimeter of the triangle

$3x - 1 = x + 9$

$2x = 10$

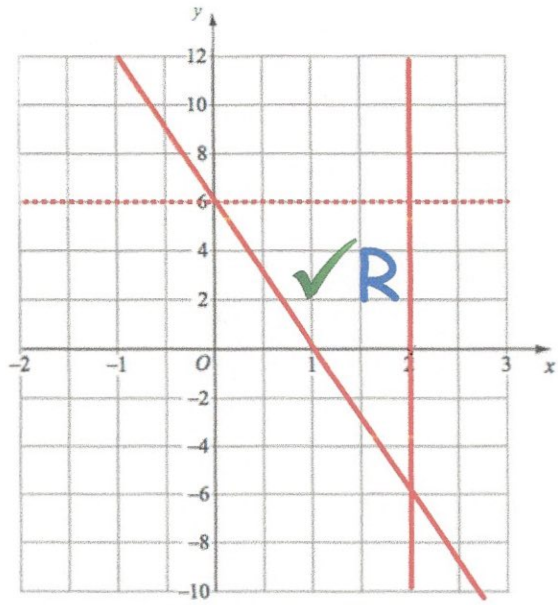
$x = 5$

34cm ....

(4)

$14 + 14 + 6 = 34$

39



$y = -6x + 6$

The region labelled R satisfies three inequalities.

$0 > -6$

State the three inequalities

$y < 6$

$x \leq 2$

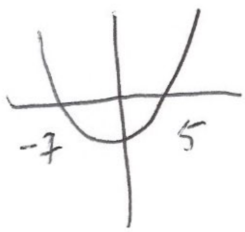
$y \geq -6x + 6$

(3)

40

Solve the inequality $x^2 + 2x - 35 > 0$

$(x+7)(x-5)$

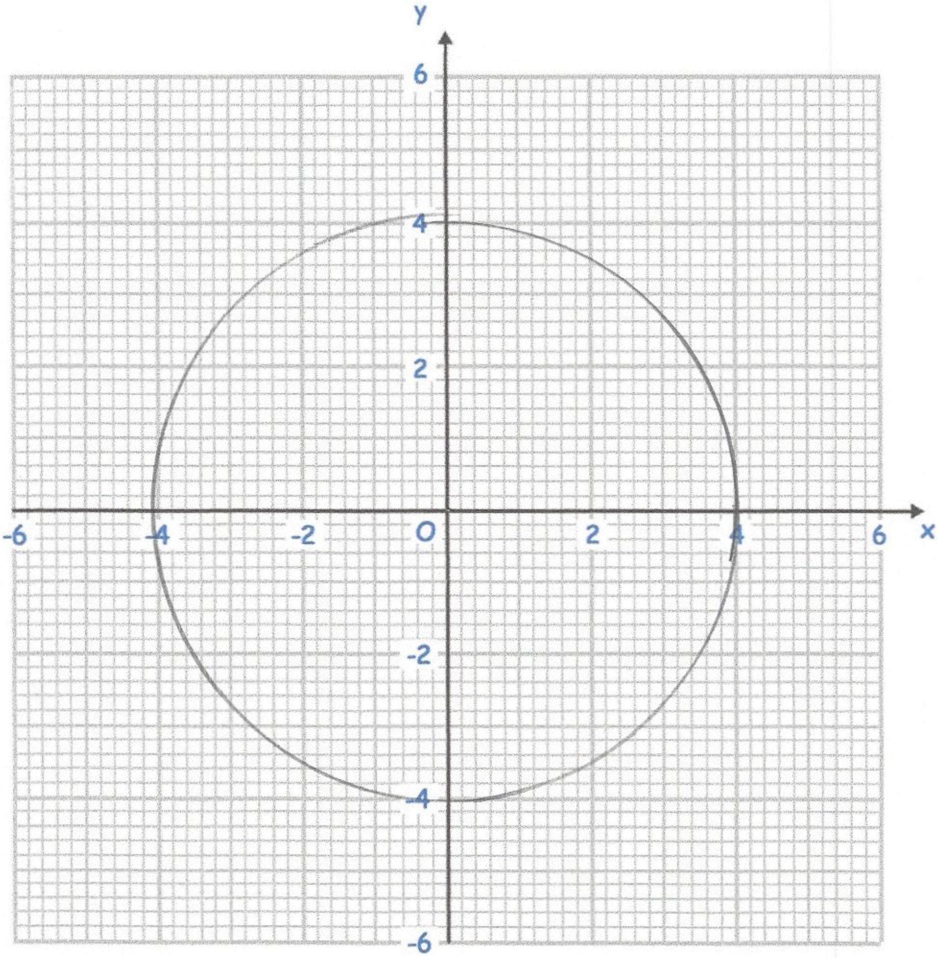


$x < -7$ or $x > 5$

(3)

40

Draw the circle with equation $x^2 + y^2 = 16$



$\sqrt{16} = 4$

$r = 4$

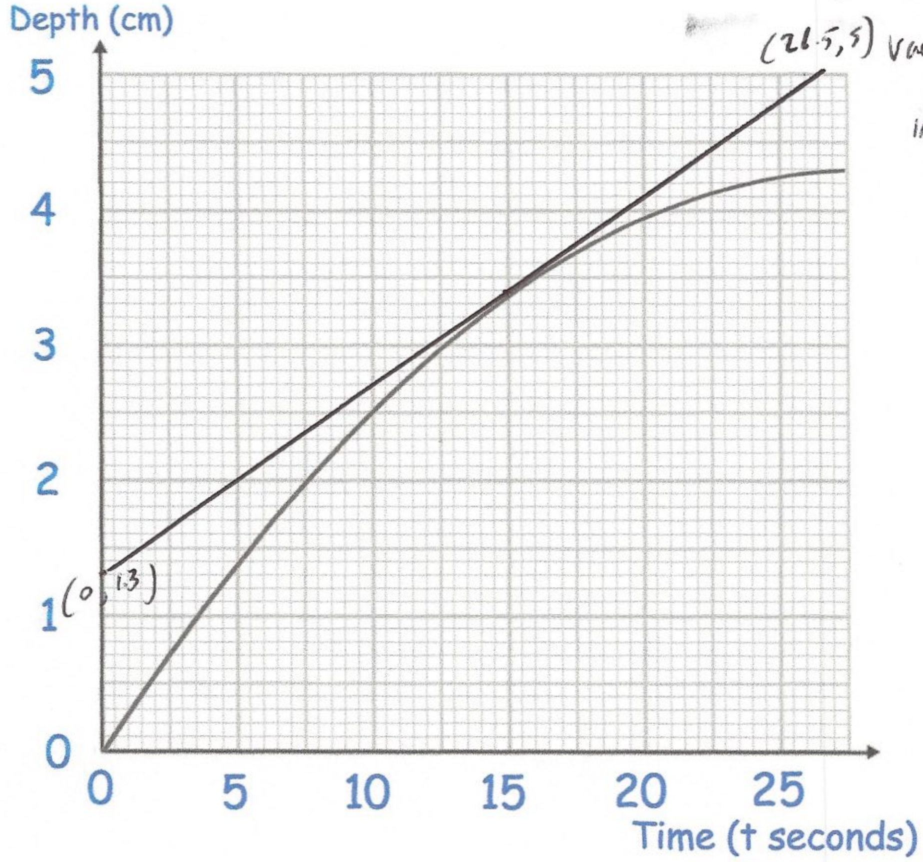
(2)

4b

Jack is filling a container with water.

The graph shows the depth of the water, in centimetres, t seconds after the start of filling the container.

* answers may vary due to individual tangents.



(a) Calculate an estimate for the gradient of the graph when t = 15 seconds.

$$\frac{\text{rise}}{\text{run}} = \frac{3.7}{26.5}$$

0.1396 (3)

(b) Describe fully what your answer to (a) represents

It is the rate at which the depth of water in the container is increasing. 0.1396 cm per second.

(2)

(c) Explain why your answer to (a) is only an estimate

It is only a hand drawn tangent - it may not be precise.

(1)

43

Solve

$$\frac{1}{x+3} - \frac{1}{x+1} = 2$$

$$\frac{x+1-(x+3)}{(x+3)(x+1)} = 2$$

$$\frac{-2}{x^2+4x+3} = 2$$

$-2 = 2x^2 + 8x + 6$

$0 = 2x^2 + 8x + 8$

$0 = x^2 + 4x + 4$

$0 = (x+2)(x+2)$

$x = -2$

....

(5)

44

The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 8 - 3x$$
$$g(x) = 4x$$

(a) Calculate the value of $gf(3)$

$f(3) = 8 - (3 \times 3) = -1$

$g(-1) = 4 \times -1$

-4

(2)

(b) Find $f^{-1}(x)$

$y = \cancel{\ } 8 - 3x$

$3x + y = 8$

$3x = 8 - y$

$x = \frac{8-y}{3}$

$f^{-1}(x) = \frac{8-x}{3}$

(2)

45

(a) Show that the equation $x^3 + 2x = 1$ has a solution between $x = 0$ and $x = 1$

$$x^3 + 2x - 1 = 0$$

when $x = 0$ $\quad 0^3 + 2 \times 0 - 1 = -1$

$x = 1$ $\quad 1^3 + 2 \times 1 - 1 = 2$

Since there is a change in sign between $x = 0$ & $x = 1$ there is a solution.

(2)

(b) Show that the equation $x^3 + 2x = 1$ can be rearranged to give $x = \frac{1}{2} - \frac{x^3}{2}$

$$2x = 1 - x^3$$

$$x = \frac{1}{2} - \frac{x^3}{2}$$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{2} - \frac{x_n^3}{2}$ twice to find an estimate for the solution of $x^3 + 2x = 1$

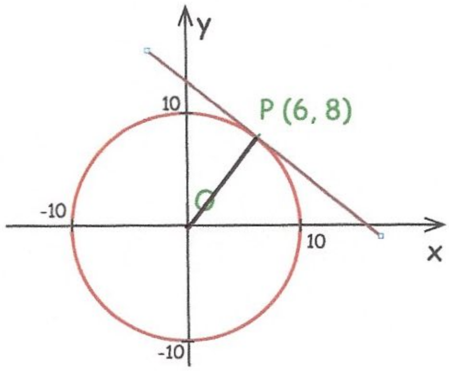
$$x_1 = \frac{1}{2} - \frac{0^3}{2} = 0.5$$

$$x_2 = \frac{1}{2} - \frac{0.5^3}{2} = \underline{0.4375}$$

(3)

48

Here is a circle, centre O, and the tangent to the circle at the point (6, 8).



Find the equation of the tangent at the point P.

gradient of OP = $4/3$

$y = -\frac{3}{4}x + c$

$8 = -4.5 + c$

$c = 12.5$

$y = -\frac{3}{4}x + \frac{25}{2}$ ....

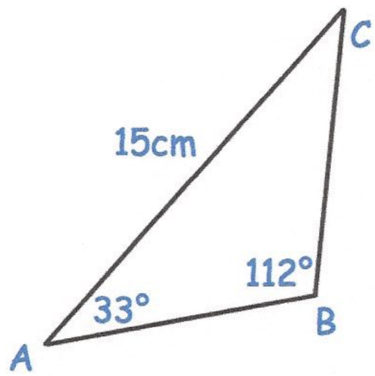
(4)

or

$y = -0.75x + 12.5$

4

(a)



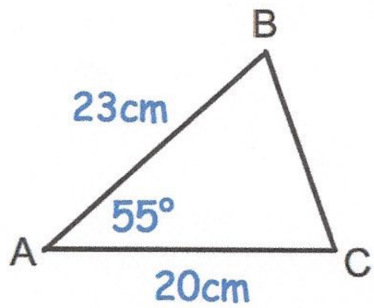
In triangle ABC the length of AC is 15cm.
Angle ABC = 112°
Angle BAC = 33°

Work out the length of BC.

$$\frac{x}{\sin 33} = \frac{15}{\sin 112}$$

8.81 ....cm
(3)

(b)



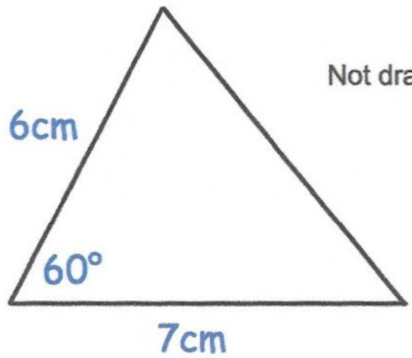
Calculate the length of BC.

$$x^2 = 23^2 + 20^2 - 2 \times 20 \times 23 \times \cos 55$$

$$x^2 = 401.3\ldots$$

20.03 ....cm
(3)

48



Not drawn is scale.

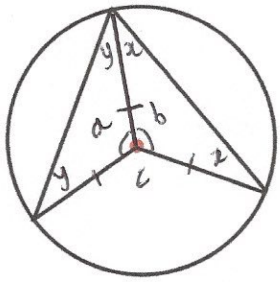
Calculate the area of the triangle.

$\frac{1}{2} \times 6 \times 7 \times \sin 60$

18.19 ....cm²

(2)

AKK
49



Prove that the angle at the centre is twice the angle at the circumference.

$a = 180 - 2y$ (angles in a triangle)

$b = 180 - 2x$ " "

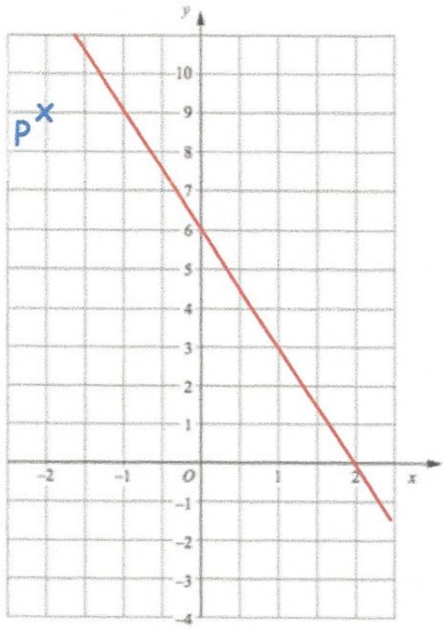
$c = 360 - (360 - 2x - 2y)$

$c = 2x + 2y$

$\therefore$ angle at centre is twice the angle at the circumference.

50

(a)



(i) Find the equation of L.

$y = -3x + 6$
(3)

The point P has coordinates (–2, 9).

(ii) Find an equation of the line that is parallel to L and passes through P.

$y = -3x + 3$
(2)

(b) The straight line K has equation y = 2x – 5
The straight line J is perpendicular to line K and passes through the point (–4, 8).

Find the equation of line J

$y = -\frac{1}{2}x + c$

$y = 8 \quad x = -4$

$8 = -\frac{1}{2}(-4) + c \qquad c = 6$

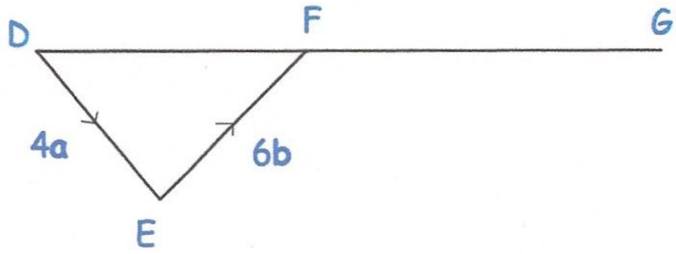
$8 = 2 + c$

$y = -\frac{1}{2}x + 6$
(3)

A8
51

DFG is a straight line.

$\overrightarrow{DE} = 4\mathbf{a}$ and $\overrightarrow{EF} = 6\mathbf{b}$



(a) Write down the vector $\overrightarrow{DF}$ in terms of **a** and **b**

$4\underline{a} + 6\underline{b}$

(1)

(b) DF : FG = 2:3

Work out the vector $\overrightarrow{DG}$ in terms of **a** and **b**
Give your answer in its simplest form.

$(4\underline{a} + 6\underline{b}) \div 2 = 2\underline{a} + 3\underline{b}$

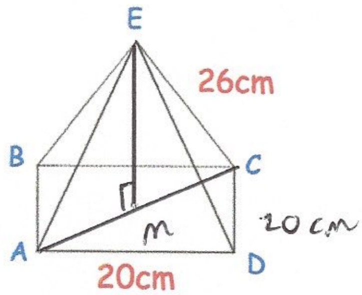
$(2\underline{a} + 3\underline{b}) \times 5$

$10\underline{a} + 15\underline{b}$

(2)

M8
52

Shown below is a square based pyramid.
The apex E is directly over the centre of the base.



AD = 20cm
CE = 26cm

(a) Work out the length of AC

$20^2 + 20^2$

$= 400 + 400$

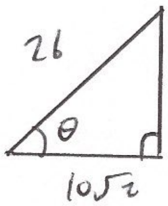
$= 800$

$AC = \sqrt{800}$

$AC = 20\sqrt{2}$

28.3 to 1dp ....cm
(2)

(b) Calculate angle CAE



$AM = 10\sqrt{2}$

$\cos\theta = \dfrac{10\sqrt{2}}{26}$

$\theta = 57.0485$

57.05 to 2dp ....°
(2)

(c) Work out the height of the pyramid

$Em = (\sin(57.0485...)) \times 26$

$= 21.817$

21.82 to 2dp ....cm
(2)

(d) Calculate the volume of the pyramid

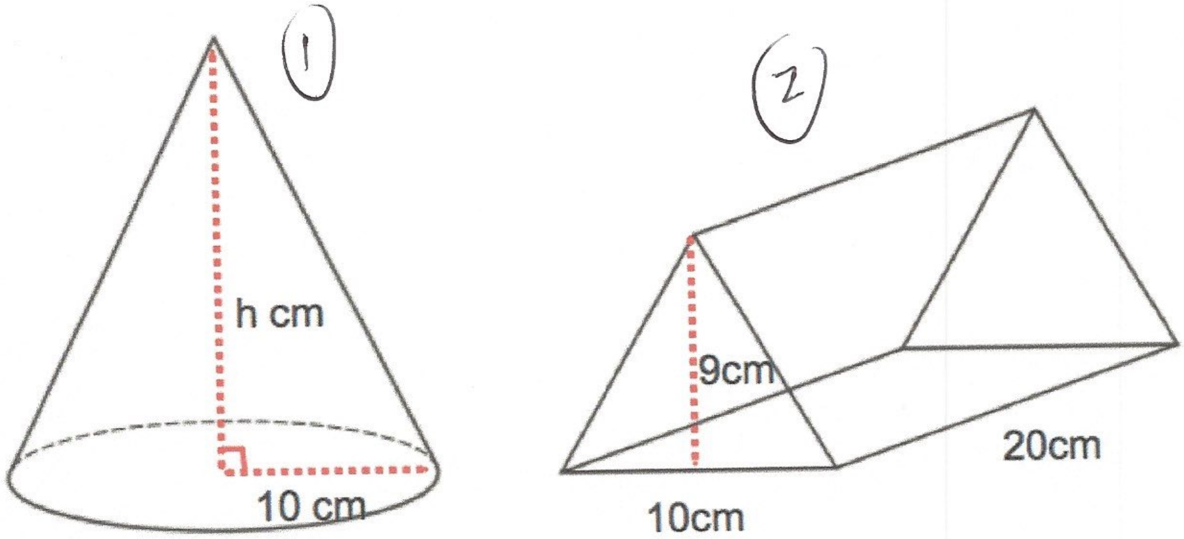
$V = \frac{1}{3} Ah$

$= \frac{1}{3} \times 20^2 \times 21.817...$

2908.99 to 2dp ....cm³
(2)

A9
53

Shown is a cone and a triangular prism.



Both solids have the same volume.

Calculate the height of the cone.

2 $v = \frac{1}{2} b h l$

$= \frac{1}{2} \times 10 \times 9 \times 20$

$= 900 \text{ cm}^3$

1 $v = \frac{1}{3} \times \pi \times r^2 \times h$

$900 = \frac{1}{3} \times \pi \times 10^2 \times h$

$2700 = \pi \times 100 \times h$

$27 = \pi \times h$

$h = 8.59\ldots$

8.6

~~8.59~~ ....cm

(3)

S4

There are 8 sweets in a bag.
Three sweets are red, three sweets are blue and two sweets are green.

Three sweets are selected at random **without** replacement.

Calculate the probability that the sweets are **not** all the same colour.

$P(RRR) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} = \frac{1}{56}$

$P(BBB) = \frac{1}{56}$

Not same

$1 - \frac{2}{56} = \frac{54}{56}$

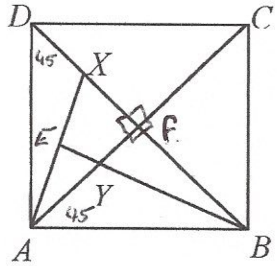
$\frac{27}{28}$

(4)

2014

55

ABCD is a square, X is a point in the diagonal BD and the perpendicular from B to AX meets AC in Y.



Prove that triangles AXD and AYB are congruent.

$AB = AD$ (square)

$\angle BAC = \angle ADB = 45^\circ$ (diagonals bisect right angle)

Let $\angle ABY = x$

$\angle AYB = 135 - x$

$\angle EYF = \angle AYB$ (vertically opposite)

$\angle AEB = \angle XEB = 90^\circ$

(4)

$XEYF$ is a kite since $\angle XEY = \angle XFY = 90^\circ$

So $\angle EXF = 45 + x$

So $\angle DXA = 135 - x$ (angles - straight line add to 180°)

As angles in ΔAXD add to 180°, $\angle DAX = x$

$\therefore$ ΔAYB is congruent to ΔAXD due to ASA

56

Prove $(2n + 9)^2 - (2n + 5)^2$ is always a multiple of 4

$(2n + 9)^2 = 4n^2 + 36n + 81$

$(2n + 5)^2 = 4n^2 + 20n + 25$

$16n + 56$

$4(4n + 14)$

$\therefore$ multiple of 4

(4)

A05

57

Solve the simultaneous equations

$2x + y = 5$
$2x^2 + y^2 = 11$

$y = 5 - 2x$

$2x^2 + (5-2x)(5-2x) = 11$

$2x^2 + (25 - 20x + 4x^2) = 11$

$6x^2 - 20x + 14 = 0$

$3x^2 - 10x + 7 = 0$

$(x-1)(3x-7) = 0$

$x = 1$ or $x = \frac{7}{3}$

$y = 3$ or $y = \frac{1}{3}$

$x = 1, y = 3$

or $x = \frac{7}{3}, y = \frac{1}{3}$